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★**Thermoelasticity with finite wave speeds.**

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The book provides a very detailed classical analysis of heat conduction in deformable solids when the processes are sufficiently fast (or 'ultra-fast', such as in laser pulses) so that finite speed of heat propagation has to be taken into account. This may be done by introducing one or two relaxation times, following the Lord-Shulman (L-S) or the Green-Lindsay (G-L) theory, respectively. In the first case, the equation for the temperature changes from the usual parabolic equation to a hyperbolic one. Then, the linearized thermoelastic system becomes hyperbolic. In the second case, the heat equation may be hyperbolic, parabolic (when the two relaxation times are equal) or elliptic.

The book is aimed at researchers in the field, and assumes that the reader is familiar with standard mathematical thermoelasticity. However, there is no need for knowledge of Sobolev spaces and the mathematical tools needed for weak formulations and solutions. This should make the book accessible to advanced graduate students with a good background in classical partial differential equations (PDEs).

The book has an extensive list of references, and many of the results in the literature are mentioned, some explained in detail, and new results are presented, too.

Since the main development is mathematical analysis, some applications of the theory are given in Chapter 12. Readers who are interested in applications, or need some motivation for this material, are advised to start there.

The first chapter provides a short review of the classical theory of thermoelasticity, and then presents the equations of the L-S and the G-L models, with one and two relaxation times, respectively.

The presentation in the book proceeds from the basics to the more advanced material. Chapters 2–9 deal with the mathematical development of the theory of linear hyperbolic thermoelasticity for the L-S models and the analysis of the G-L models. The emphasis is on the derivation of the systems of equations and their thermodynamic correctness. The formulations of the problems as initial-boundary value problems include the displacement-temperature, the stress-temperature, and the stress-heatflux systems. The existence of solutions to these systems follows from the general existence theorems for hyperbolic PDEs. Uniqueness is proved by using energy conservation and energy estimates. Then, theorems on the domains of influence are proven. The convolutional variational principles are derived. Fundamental solutions of symmetric problems are shown. Integral representations and integral equation formulations of the problems are provided. And a derivation of thermoelastic polynomials is given.

The propagation of singular surfaces and planar shock waves in thermoelasticity are studied in detail in Chapter 10, and the following chapter deals with periodic solutions (in time) which are analyzed for planar, spherical, and cylindrical waves.

Chapter 12, as noted above, deals with some physical aspects and various applications of the theory. First, the authors show how a general thermodynamic approach with internal variables leads to the Maxwell-Cattaneo heat equation with one relaxation time. They also describe how one can extend the dynamic helix model, which possibly includes periodic cells, to include fast thermal conduction. Some discussion of surface waves is presented, as well as results on thermoelastic damping in nano-mechanical resonators. A short discussion of the so-called “anomalous diffusion“ that is governed by fractional time derivatives is provided, too. The chapter is concluded with a brief description of fast thermal diffusion in fractal media.

The final chapter provides a short description of a 1D hyperbolic rigid heat conductor of Coleman type. The authors, using thermodynamics, obtain a 1D nonlinear system for the temperature and the heat flux. Then, they derive a closed-form solution and traveling-wave solutions and, finally, they apply asymptotics of the weakly-nonlinear geometric optics type to the 1D system and show that small initial data with high frequency can cause a pair of thermal shock waves.

The book has a very classical flavor concerning PDEs, is interesting, contains substantial coverage of very recent results, and should not only be a textbook for advanced students of the subject, but also serve as a reference book to those who are involved in research of theoretical or applied issues related to thermoelastic systems with finite speeds of heat propagation.

Reviewed by *Meir Shillor*