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Thermoelasticity with finite wave speeds. (English)

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The book focuses on two theories of hyperbolic thermoelasticity, that of Lord-Shulman and that of Green-Lindsay. The main goal of the book is to present some mathematical results concerning these sets of evolution equations. The book gathers results available in the literature and presents some possible extensions of such theories.

In the first chapter, the authors set the systems of equations they will study in the rest of the book. They start with a general description of a thermoelastic process which consists of the linear constitutive equation $E_{ij} = (u_{i,j} + u_{j,i})/2$, of the dynamic equilibrium equation $S_{ij,j} + b_i = \rho\ddot{u}_i$, of constitutive equations $S_{ij} = C_{ijkl}E_{kl} + M_{ij}\vartheta$, $\theta_0\eta = -\theta_0 M_{ij}E_{ij} + C_E\vartheta$, of an energy conservation law $\theta_0\dot{\eta} = -q_{i,i} + r$ and of a dissipative relation $q_i = -k_{ij}\vartheta_{,j}$. Here u represents the displacement field within the elastic body, E is the strain tensor, S is the stress tensor, q is the heat flux, θ is the absolute temperature and $\vartheta = \theta - \theta_0$. They write equivalent formulations of this thermoelastic problem. They then present the case involving one relaxation time, replacing the equation $q_i = -k_{ij}\vartheta_{,j}$ by $Lq_i = -k_{ij}\vartheta_{,j}$, where the operator L is taken as $1 + t_0\partial/\partial t$, for some $t_0 > 0$ which is the relaxation time. They describe several properties of this model, called L-S model, being first studied by Lord and Shulman in 1967. The rest of this first chapter is devoted to the case where two relaxation times are introduced when modifying the four first equations of the above problem. In this case, the authors again derive several properties of this model, called G-L model, which eliminates the paradox of infinite speeds of propagation and which has been studied by Green and Lindsay in 1972.

In chapter 2, the authors introduce the boundary value conditions and the initial conditions for a thermoelastic problem. The boundary ∂B of the material is divided in two ways $\partial B = \partial B_1 \cup \partial B_2 = \partial B_3 \cup \partial B_4$. In the L-S model, four mixed displacement-temperature boundary conditions are imposed. Four initial conditions are imposed (for the displacement field and for the temperature and for their first-order time derivative). In this L-S theory, the authors also consider a mixed stress-heat-flux problem. In the G-L theory, the authors consider either a mixed displacement-temperature problem or a mixed stress-temperature problem.

Chapter 3 is devoted to existence and uniqueness results. In each of the four problems described in the preceding chapter, the authors first prove the existence of at most one solution. They finally prove an existence result for a conventional thermoelastic problem, through the notion of generalized sequences (of classical solutions).

In chapter 4, the authors introduce the notion of thermoelastic disturbance, as the solution of some thermoelastic problem with some thermoelastic process. They prove that the thermoelastic disturbances have a character of wave propagating in the material with finite speeds. In each of the above four cases, they define the notion of domain of influence and characterize this domain of influence in terms of properties of smooth solutions of the corresponding model. This is obtained using estimates obtained when integrating the equations of the corresponding model.

The purpose of the fifth chapter is to establish variational formulations of the thermoelastic L-S or G-L models. The authors use equivalent equations of these models and the Laplace transform.

In chapter 6, the authors derive what they call a central equation in the L-S or G-L theories. This consists to take vanishing body forces. Eliminating ϑ in each case and combining the field equations, they obtain a single equation with a (central) operator Γ. The authors prove that the central operators of the G-L and L-S theories coincide if the relaxation times are equal. Considering the G-L theory, the main result of this chapter proves that a solution ϕ of the equation $\Gamma\phi = 0$ in $B \times (0, \infty)$, where Γ is the central operator of the G-L theory (of order 4 with respect to the space variables) and satisfying $\partial^k\phi/\partial t^k(.,0) = 0$ on B, can be decomposed as the sum of two functions ϕ_1 and ϕ_2, each of them being the solution of a pde of second order with respect to the space variables and which involves a convolution operation with a kernel associated to the Bessel function J_1 and some physical quantities. The rest of chapter 6 is devoted to the description of some properties of these physical quantities and of the convolution kernel.

In chapter 7, the authors describe aperiodic in time solutions of the G-L theory. They start with the presentation of the fundamental solutions in the case of the potential-temperature problem. Considering the problems $\widehat{L}_i\phi_i = 0$ $(i = 1, 2)$ in $B \times [0, \infty)$, where the operators $\widehat{L}_i$ have been introduced in the preceding chapter, they add the initial conditions $\phi_i(., 0) = \dot{\phi}_i(., 0) = 0$ and the boundary conditions $\phi_i = f_i$ on $\partial B \times [0, \infty)$. After some transformation of the above problems, the authors look for the first solution ϕ_1 as $\sum_j T_{1j}\varphi_j$ where the φ_j are the eigenfunctions of the operator $-\nabla^2$ in B, with homogeneous Dirichlet boundary conditions on ∂B. Of course this structure is interesting if the φ_j are known. The section of this chapter presents some computations which give the structure of the T_{1j} in terms of the kernel G_k defined as $G * G * \cdots * G$ (k times), with $G(t) = 2e^{\alpha t}J_1(\beta t)/\beta t$. The authors here use the Laplace transform and derive some polynomials called polynomials of thermoelasticity. Similar computations are given for the second fundamental solution ϕ_2. From these expressions, the authors first deduce the structure of the solution of a potential-temperature problem for a 3D bounded domain. They then deduce the structure of the solution of a 1D thermoelastic G-L problem Then the authors move to the presentation of a solution of Nowacki type, that is two aperiodic solutions of a G-L theory corresponding to a smooth thermomechanical loading. They indeed prove that a displacement-temperature wave of Nowacki type is the superposition of two spherical wave having a strong discontinuity of order -1. Coming back to the 1D situation, the authors present a solution of Danilovskaya type. Again such a solution is proved to be the superposition of two plane waves of strong discontinuity of order -1. The chapter 7 ends with the construction of a thermoelastic response of a half-space to laser irradiation. In each case, the authors present the computations which lead to the expression of the solution of the problem under consideration with lengthy details.

In chapter 8, the authors present integral representations of the fundamental solutions in the G-L theory. They first compute the Green functions of the corresponding problems

using the Laplace transform and its inverse. This leads to an integral representation of the fundamental solutions. A similar approach is developed for the solution of a central system of equations. In the last section of this chapter, the authors consider a potential-temperature problem within this G-L theory. They study the inner and the outer problems and derive some integral equations of Fredholm type. They also prove some properties of the kernels which are here involved.
In the short chapter 9, the authors come back to the structure of the fundamental solutions in the G-L theory which has been obtained in chapter 7 and which involves the polynomials of thermoelasticity. They here establish some recurrence relations between these polynomials. They also build some ordinary differential equations associated to these polynomials and derive some integral relations satisfied by these polynomials.
In chapter 10, the authors consider singular surfaces propagating in a thermoelastic medium, recalling that in chapter 7 the solutions of Nowacki or Danilovskaya types involve such singular surfaces or order -1. They first give several definitions of singular surfaces in the L-S or G-L theories and for different kinds of thermoelastic problems. They then focus on the propagation of a plane shock wave in a thermoelastic half-space with one relaxation time or that of a plane acceleration wave in a thermoelastic half-space with two relaxation times.
Chapter 11 is devoted to the study of time-periodic solutions in the L-S or G-L theories. The authors start with the 1D displacement-temperature field equations with two relaxation times. They here look for time-periodic solutions and draw some computations and approximations on the expression of the solution. A second problem is then considered which deals with spherical (resp. cylindrical) waves in an infinite thermoelastic body with two relaxation times and assuming a spherical (resp. cylindrical) symmetry. Again looking for periodic solutions the authors draw some computations of the solution.
In the short chapter 12, the authors describe some different approaches concerning hyperbolic thermoelasticity. They start presenting the theory developed by Green and Naghdi. They study the properties of the Maxwell-Cattaneo equation. They also describe some applications of the L-S or of the G-L theories to helices and chiral media, to surface waves in an isotropic homogeneous half-space and to thermoelastic damping in nanomechanical resonators. The chapter ends with an observation on an anomalous heat equation involving time fractional derivatives.
In the final chapter 13, the authors describe a rigid but nonlinear hyperbolic heat conductor of Coleman type. They first write the field equations in the 1D case. After some computations, they present the closed-form solutions and present some properties of the travelling wave solutions. This leads to an asymptotic analysis of the solution of a nonlinear hyperbolic conduction model with a weakly perturbed initial condition.
Throughout the whole book, the authors do not hesitate to present complete computations on their models or examples. They also illustrate their computations with figures.

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∗80-02 Research monographs (classical thermodynamics)

80A17 Thermodynamics of continua
80M30 Variational methods